Mastermind Study Material

COORDINATE GEOMETRY

3.1 INTRODUCTION:

If we choose x = – 1, 0 etc. from the set R of the real numbers and the corresponding values of y obtained from the relation $y = 3x - 2$

(i) $x = -1 \Rightarrow y = 3(-1) - 2 = -5$ (ii) $x = 0 \Rightarrow y = 3(0) - 2 = -2$

i.e., –5, –2 are called dependent variables while – 1, 0 are called independent variables. Now the pairs (–1, –5), (0, –2) are known as ordered pairs and coordinates of two different points in the Cartesian plane.

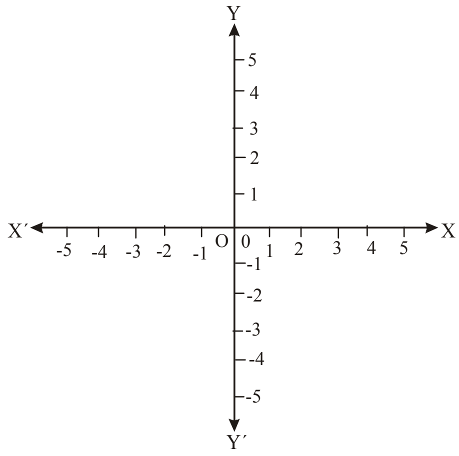
Ordered Pair : A pair of numbers a and b listed in a specific order with a at the first place and b at the second place is called an ordered pair (a, b).

Note that $(a, b) \neq (b, a)$.

3.2 RECTANGULAR COORDINATE AXES:

Let X'OX and Y'OY be two mutually perpendicular lines through any point O in the plane of the paper. This point O, is called **origin.**

Now choose a convenient unit of length and starting from the origin as zero, mark off a number scale on the horizontal line X'OX, positive to the right of the origin O and negative to the left of origin O. Also, mark off the same scale on the vertical line Y'OY, positive upwards and negative downwards of the origin O.



The line X'OX is called the **x-axis or axis of x**.

The line Y'OY is known as the **y-axis or axis of y**.

The x-axis and y-axis taken together are called the co-ordinate axis or the axes of coordinates.

Illustration 1. Fill in the blanks:

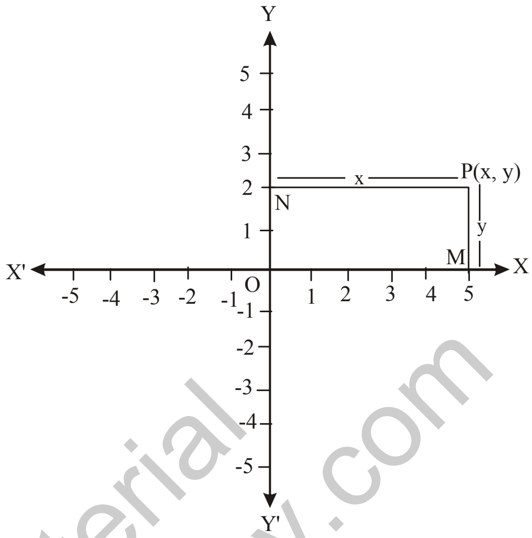
(i) The point in the positive direction to the right of origin at a distance of '4' units from origin lies on __________.

(ii) The point at a distance of 'x' units above the origin lies on __________.

Solution : (i) positive x-axis, (ii) positive y-axis

3.3 CARTESIAN COORDINATES OF A POINT:

Let X'OX and Y'OY be the coordinate axes, and let P be any point in the plane. Draw perpendiculars PM and PN from P on x and y axis respectively. The length of the line segment OM in the units of scale chosen is called **the x-coordinate or abscissa of point P**.

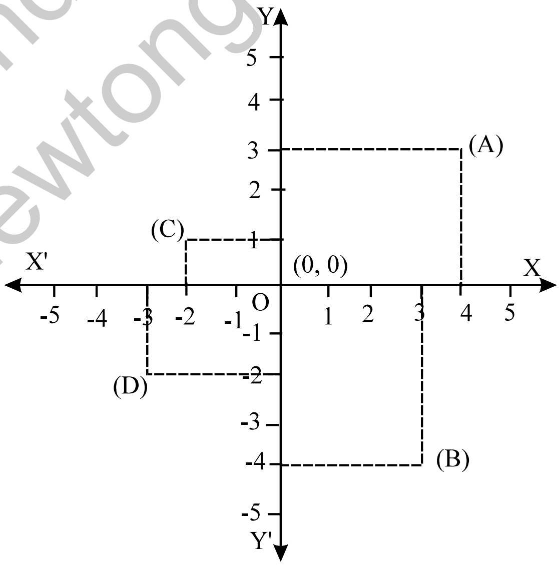


Similarly, the length of the directed line segment ON on the same scale is called **the y-coordinate or ordinate of point P**. If OM = x and ON = y, then the position of the point P in the plane with respect to the coordinate axes is represented by the ordered pair (x, y). **The ordered pair (x, y) is called the coordinates of point P**. Thus, for a given point, the abscissa and ordinate are the distances of the given point from y-axis and x-axis respectively.

Illustration 2. See figure & complete the following statements:

(i) The abscissa and the ordinate of the point A are ___ and ___, respectively. Hence, the coordinates of A are (___,__).

(ii) The abscissa and the ordinate of the point B are ___ and ___, respectively. Hence the coordinates of B are (___, ___).



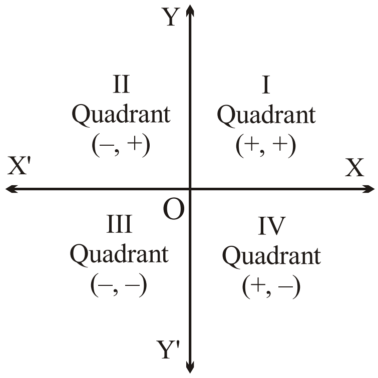
Solution :

(i) 4, 3, (4, 3) (ii) 3, – 4, (3, – 4)

3.4 QUADRANTS :

The x-axis and y-axis divide the cartesian plane into four regions, called the **quadrants**.

Let X'OX and Y'OY be the coordinate axes, which divide the plane in four quadrants. The regions XOY, X'OY and X'OY' and Y'OX are known as the first, the second, the third and the fourth quadrants respectively.



(i) Towards the right side of the Y-axis, x-coordinate of any point on the graph paper is taken positive and towards the left side of the Y-axis, x-coordinate is taken negative.

(ii) Above the X-axis, the y-coordinate of any point on the graph paper is taken positive and below the x-axis, y-coordinate is taken negative.

The four quadrants are characterised by the following signs of abscissa and ordinate:

Quadrant	x coordinate	y coordinate	Point
First Quadrant	+	+	(+, +)
Second Quadrant	–	+	(–, +)
Third Quadrant	–	–	(–, –)
Fourth Quadrant	+	–	(+, –)

(i) The coordinates of the origin are (0, 0).

(ii) The coordinates of any point on x axis are of the form (x, 0).

(iii) The coordinates of any point on y axis are of the form (0, y).

(iv) If the abscissa of a point is zero, it would lie on the y axis and if its ordinate is zero it would lie on x-axis.

Illustration 3.

In which quadrant or on which axis are the points (–4, 6), (5, –2), (–7, 0) and (–2, –1) lie?

Solution:

(i) ∵ x coordinate < 0, y coordinate > 0, point (–4, 6) lies in the II quadrant.

(ii) ∵ x coordinate > 0, y coordinate < 0, point (5, –2) lies in the IV quadrant.

(iii) ∵ x coordinate < 0, y coordinate = 0, point (–7, 0) lies on x axis.

(iv) ∵ x coordinate < 0, y coordinate < 0, point (–2, –1) lies in the III quadrant.

Illustration 4.

Write the coordinates of a point lying on x-axis to the left of origin at a distance of 2 units.

Solution: (–2, 0)

Illustration 5.

Write the coordinates of a point lying on y-axis at a distance of 5 units above origin.

Solution: (0, 5)

3.5 PLOTTING OF POINTS IN THE CARTESIAN PLANE :

In order to plot the points in a plane, we may use the following algorithm:

(i) Draw two mutually perpendicular lines on the graph paper, one horizontal and other vertical.

(ii Mark their intersection point as O (origin). The horizontal line as X'OX and the vertical line as Y'OY. The X'OX is the x-axis and the line Y'OY is the y-axis.

(iii) Choose a suitable scale on x-axis and y-axis and mark the points on both the axes.

(iv) Obtain the coordinates of the point which is to be plotted. Let the point be P(a, b). To plot this point, start from the origin and move '| a |' units along OX or OX' according as 'a' is positive or negative. Suppose we arrive at point M. From point M move vertically upward or downward through | b | units according as b is positive or negative. The point where we arrive finally is the required point P(a, b).

Illustration 6.

Plot point A(3, 4) on a graph paper.

Solution:

(i) Let X'OX and Y'OY be the coordinate axes.

(ii) Let 1 column on x axis and y axis is equal to 1 units.

(iii) Start from the origin and move 3 units along OX and mark this point as M.

(iv) From M move 4 units vertically upward.

(v) This point is the required point A(3, 4).

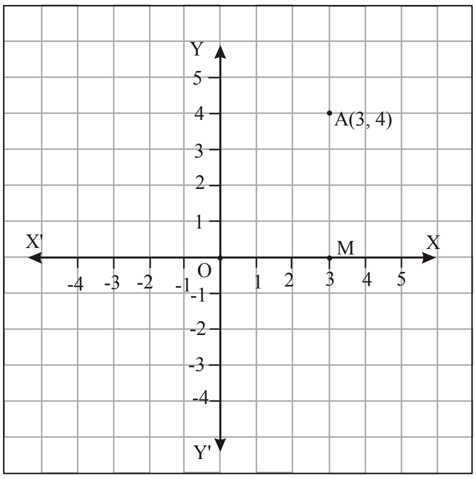


Illustration 7.

Draw the lines X'OX and YOY' as axes on the plane of a paper and plot the points given below:

(i) A(5, 3) (ii) B(–3, 2) (iii) C(–5, – 4) (iv) D(2, –6)

Solution:

Let X'OX and YOY' be the coordinate axes.

Fix a convenient unit of length and starting from O, mark equal distances on OX, OX', OY and OY':

(i) Starting from O, take +5 units on the x-axis and then +3 units on the y-axis to obtain the point A(5, 3).

(ii) Starting from O, take –3 units on the x-axis and then +2 units on the y-axis to obtain the point B(–3, 2).

(iii) Starting from O, take –5 units on the x-axis and then –4 units on the y-axis to obtain the point B(–5, –4).

(iv) Starting from O, take 2 units on the x-axis and then –6 units on the y-axis to obtain the point D(2, –6).

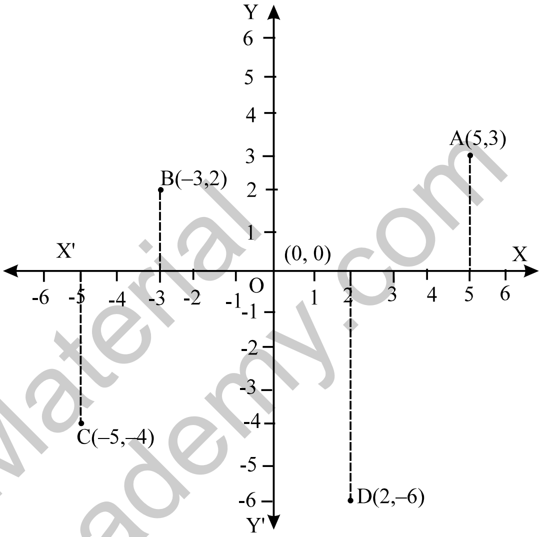


Illustration 8.

Draw the quadrilateral whose vertices are:

(–2, –2), (–4, 2), (–6, –2) and (–4, –6).

Solution:

With rectangular axes, plot the points A(–2, –2), B(–4, 2),C(–6, –2) and D(–4, –6). Join A to B, B to C, C to D and D to A. The quadrilateral so formed is a **Rhombus ABCD**.

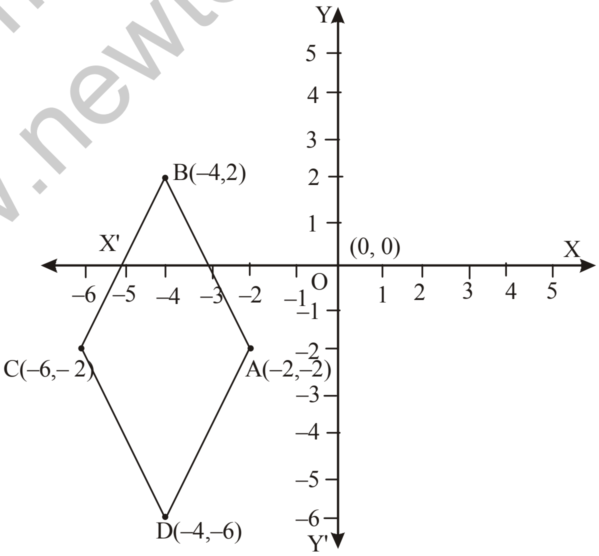


Illustration 9.

Plot points A(–6, 0) and B(0, –5) on the graph paper. Join A and B. What figure do you obtain. Find the area of the figure so formed.

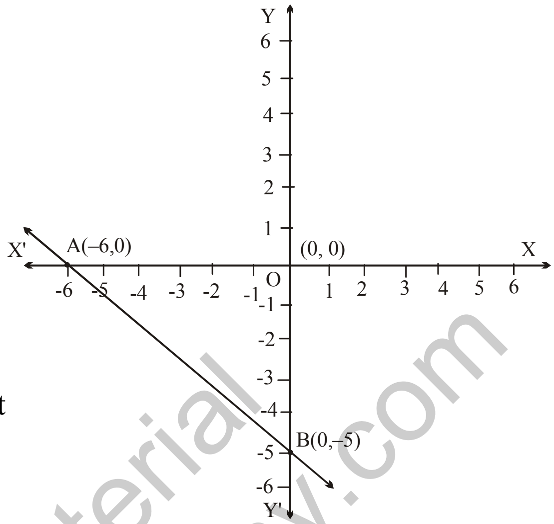
Solution:

(i) Move 6 units to the left of origin and mark that point as A.

(ii) Move 5 units below origin and mark that point as B.

(iii) Join A and B.

The figure obtained is a right angled triangle as $\angle AOB$ is 90º.



Area of triangle is given by = $\frac{1}{2}$ × base × height

Let AO be base of ΔAOB.

BO be height of ΔAOB.

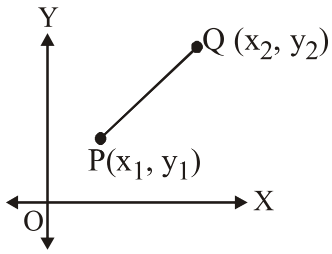
area (ΔAOB) = $\frac{1}{2} \times (6) \times (5) = 15$ square units.

3.6 DISTANCE BETWEEN TWO POINTS:

The distance between any two points in the plane is the length of the line segment joining them. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i.e. $PQ = \sqrt{(\text{difference of abscissa})^2 + (\text{difference of ordinates})^2}$



(i) The distance of a point P(x, y) from the origin O(0, 0) is given by $OP = \sqrt{x^2 + y^2}$

(ii) For three points to be collinear, prove that the sum of the distances between two pairs of points is equal to the third pair of points.

Illustration 10.

Find the distance between the two points P(–5, 7) and Q(–1, 3).

Solution:

$$PQ = \sqrt{(-1-(-5))^2 + (3-7)^2} = \sqrt{(-1+5)^2 + (-4)^2} = \sqrt{(4)^2 + (-4)^2}$$

$$PQ = \sqrt{16+16} = \sqrt{32}$$

∴ **The distance between two points P and Q is $\sqrt{32}$ units.**

APPLICATIONS OF DISTANCE FORMULA:

(i) For given three points A, B, C to decide whether they are collinear or vertices of a particular triangle. First we find the length of AB, BC and CA then we shall find that the points are

(a) Collinear, if the sum of two shorter distances is equal to the largest distance.

(b) Vertices of an equilateral triangle if AB = BC = CA.

(c) Vertices of an isosceles triangle if AB = BC or BC = CA or CA = AB

(d) Vertices of a right angled triangle if $AB^2 + BC^2 = CA^2$ etc.

(ii) For given four points A, B, C and D.

(a) AB = BC = CD = DA; AC = BD ⇒ ABCD is a square.

(b) AB = BC = CD = DA ⇒ ABCD is a rhombus.

(c) AB = CD, BC = DA, AC = BD ⇒ ABCD is a rectangle.

(d) AB = CD, BC = DA ⇒ ABCD is a parallelogram.

(iii) (a) Diagonals of square, rhombus, rectangle and parallelogram always bisect each other.

(b) Diagonals of rhombus and square bisect each other at right angle.

(c) Three given points are collinear if area of the triangle formed from these three points is zero.

(d) Four given points are collinear, if area of quadrilateral formed from these four points is zero.

Illustration 11.

Prove that the points (1, –1), $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and (1, 2) are the vertices of an isosceles triangle.

Solution

Let the point (1, –1), $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and (1, 2) be denoted by P, Q and R, respectively. Now,

$$PQ = \sqrt{\left(-\frac{1}{2}-1\right)^2 + \left(\frac{1}{2}+1\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$$

$$QR = \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$$

$$PR = \sqrt{(1-1)^2 + (2+1)^2} = \sqrt{9} = 3$$

From the above, we see that PQ = QR.

∴ The triangle is isosceles.

Illustration 12

Show that the points (– 2, 5), (3, – 4) and (7, 10) are the vertices of a right triangle.

Solution

Let the three points be A (– 2, 5), B (3, – 4) and C (7, 10).

Then, $AB^2 = (3 + 2)^2 + (-4 - 5)^2 = 106$

$BC^2 = (7 - 3)^2 + (10 + 4)^2 = 212$

$AC^2 = (7 + 2)^2 + (10 - 5)^2 = 106$

We see that

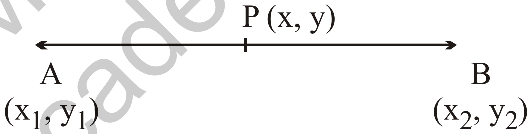
$\Rightarrow \quad BC^2 = AB^2 + AC^2 \quad \Rightarrow \quad 212 = 106 + 106 \quad \Rightarrow \quad 212 = 212$

$\therefore \quad \angle A = 90^0$.

Thus, ABC is a right triangle, right angled at A.

3.7 SECTION FORMULA :

The coordinates of the point P(x, y) which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio m : n are given by.



$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

Proof :

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be the two given points and P be a point on AB which divides it in the given ratio m : n i.e. AP : PB = m : n. We have to find the co-ordinates of P. Let coordinates of P = (x, y). Draw the perpendicular AL, PM, BN on OX and AK, PT on PM and BN respectively. Then, from similar triangles AKP and PTB, we have $\frac{AP}{PB} = \frac{AK}{PT} = \frac{PK}{BT}$ ....(i)

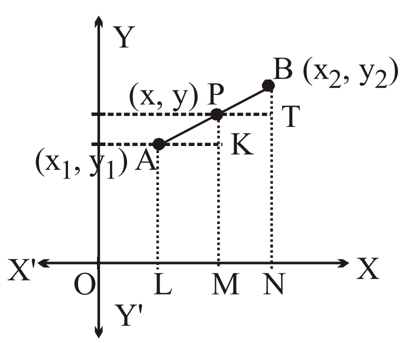
Now, $AK = LM = OM - OL = x - x_1$

$PT = MN = ON - OM = x_2 - x$

$PK = MP - MK = MP - LA = y - y_1$

$BT = NB - NT = NB - MP = y_2 - y$

From (i), we have $\frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$



The first two relations give, $\frac{m}{n} = \frac{x - x_1}{x_2 - x}$

or $\quad mx_2 - mx = nx - nx_1$

or $\quad x(m + n) = mx_2 + nx_1$

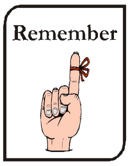
or $\quad x = \frac{mx_2 + nx_1}{m + n}$

Similarly, from the relation $\frac{AP}{PB} = \frac{PK}{BT}$, we get $\frac{m}{n} = \frac{y - y_1}{y_2 - y}$ which gives on simplification.

$$y = \frac{my_2 + ny_1}{m + n}$$

Hence, $x = \frac{mx_2 + nx_1}{m + n}$ and $y = \frac{my_2 + ny_1}{m + n}$ ....(1)

Hence, co-ordinates of a point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) in the ratio m : n internally is $\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$.



Note : (i) If P is the midpoint of AB, then it divides AB in the ratio 1 : 1, so its coordinates are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

(ii) The coordinates of the point which divides the line segment joining the points (x_1, y_1) and (x_2, y_2) externally in the ratio m : n are $\left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}\right)$.

(iii) If the midpoints of the sides BC, AC and AB of ΔABC respectively are $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, x_3)$, then its vertices are $A(-x_1 + x_2 + x_3, -y_1 + y_2 + y_3)$, $B(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$ and $C(x_1 + x_2 - x_3, y_1 + y_2 - y_3)$.

(iv) The fourth vertex of a parallelogram whose three consecutive vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) when taken in order is $(x_1 - x_2 + x_3,\ y_1 - y_2 + y_3)$.

Illustration 13

Find the mid-point of the line segment joining the points (2, – 6) and (6, – 4) and M be the mid-point of AB.

Solution:

Let A (2, – 6) and B (6, – 4) be the given points and M be the mid-point of AB.

Then, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 6}{2}, \frac{-6 + (-4)}{2}\right) = (4, -5)$

Hence, the mid-point of AB is (4, – 5).

Illustration 14

Find the coordinates of the point which divides the line segment joining the points (4, –3) and (8, 5) in the ratio 3 : 1 internally.

Solution:

Let P(x, y) be the required point. Using the section formula, we get

$m = 3 \quad x_1 = 4 \quad x_2 = 8$

$n = 1 \quad y_1 = -3 \quad y_2 = 5$

$$x = \frac{3(8)+1(4)}{3+1} = 7\ ;\ y = \frac{3(5)+1(-3)}{3+1} = 3$$

∴ (7, 3) is the required point.

Illustration 15

In what ratio does the point (–4, 6) divide the line segment joining the points A(–6, 10) and B(3, –8)?

Solution:

Let (–4, 6) divides AB internally in the ratio m : n. Using the section formula, we get

$$(-4, 6) = \left(\frac{3m-6n}{m+n}, \frac{-8m+10n}{m+n}\right)$$

We know if (x, y) = (a, b), then x = a and y = b

So, $-4 = \frac{3m-6n}{m+n}$ and $6 = \frac{-8m+10n}{m+n}$

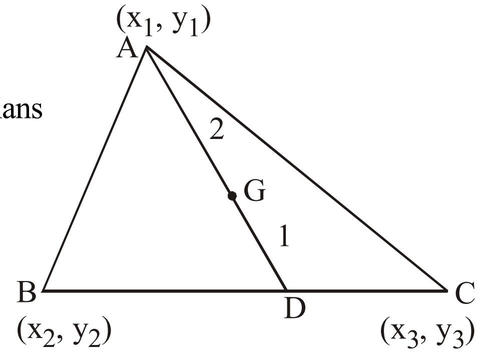
Now, $-4(m+n) = 3m - 6n \Rightarrow -4m - 4n = 3m - 6n$

$\Rightarrow -4m - 3m = -6n + 4n \Rightarrow -7m = -2n \Rightarrow 7m = 2n$

$\Rightarrow \frac{m}{n} = \frac{2}{7} \Rightarrow m : n = 2 : 7$

∴ The point (–4, 6) divides the line segment joining points A(–6, 10) and B(3, –8) in the ratio 2 : 7.

3.8 CENTROID OF A TRIANGLE:



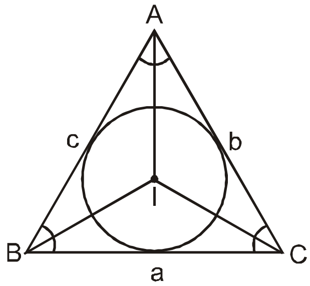
Let A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are vertices of any triangle thenthe centroid is the point of intersection of the medians (Line segment joining the mid-point of a side and its opposite vertex is called a median of the triangle).

Centroid divides the median in the ratio of 2 : 1.

Co-ordinates of centroid $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

3.9 IN-CENTRE OF A TRIANGLE:

The coordinates of the **in-centre (intersection point of angle bisector segment)** of a triangle whose vertices **(x_1, y_1), (x_2, y_2)** and **(x_3, y_3)** are $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$.



Where **a, b, c** be the lengths of the sides **BC, CA, AB** respectively.

NOTE :

(i) Incentre divides the angle bisectors in the ratio, $(b+c) : a$; $(c+a) : b$ & $(a+b) : c$.

(ii) Orthocenter, Centroid & Circumcenter are always collinear & centroid divides the line joining orthocenter & circumcenter in the ratio 2 : 1 respectively.

(iii) In an isosceles triangle Centrod (G), Orthocenter (O), Incenter (I) & Circumcenter (C) lie on the same line and in an equilateral triangle, all these four points coincide.

Illustration 16

Find the coordinates of (i) centroid (ii) in-centre of the triangle whose vertices are (0, 6), (8, 12) and (8, 0).

Solution

(i) We know that the coordinates of the centroid of a triangle whose angular points are (x_1, y_1), (x_2, y_2) (x_3, y_3) are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

So the coordinates of the centroid of a triangle whose vertices are (0, 6), (8, 12) and (8, 0) are $\left(\frac{0+8+8}{3}, \frac{6+12+0}{3}\right)$ or $\left(\frac{16}{3}, 6\right)$.

(ii) Let A (0, 6), B (8, 12) and C (8, 0) be the vertices of triangle ABC.

Then $c = AB = \sqrt{(0-8)^2+(6-12)^2} = 10$, $b = CA = \sqrt{(0-8)^2+(6-0)^2} = 10$

And $a = BC = \sqrt{(8-8)^2+(12-0)^2} = 12$.

The coordinates of the in-centre are

$$\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

or $\left(\frac{12\times0+10\times8+10\times8}{12+10+10}, \frac{12\times6+10\times12+10\times0}{12+10+10}\right)$ or $\left(\frac{160}{32}, \frac{192}{32}\right)$ or (5, 6).

Illustration 17

Find the centroid of ΔABC whose vertices are A (2, – 3), B (4, 2) and C (– 3, – 2).

Solution:

Given, A (2, – 3), B (4, 2) and C (– 3, – 2).

So, centroid of $\Delta ABC = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$$= \left(\frac{2+4-3}{3}, \frac{-3+2-2}{3}\right) = (1, -1)$$

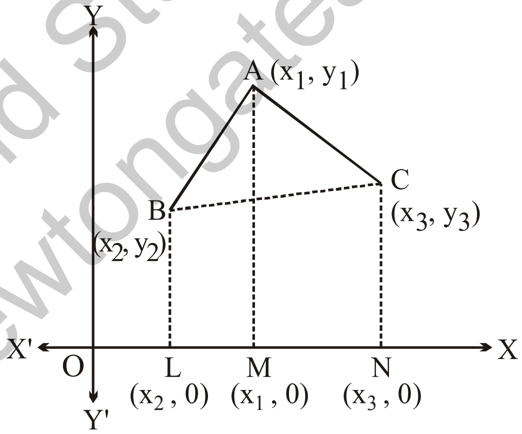
Hence, (1, – 1) is the centroid of ΔABC.

3.10 AREA OF A TRIANGLE :

Let ABC be any triangle whose vertices are A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3). Draw BL, AM and CN perpendiculars from B, A and C respectively on the x-axis. ABLM, AMNC and BLNC are all trapeziums.

Area of ΔABC = Area of trapezium ABLM + Area of trapezium AMNC – Area of trapezium BLNC

We know that, Area of trapezium = $\frac{1}{2}$ × (sum of parallel sides) × (distance between them)



$\therefore$ Area of $\Delta ABC = \frac{1}{2}(BL + AM)(LM) + \frac{1}{2}(AM + CN)(MN) - \frac{1}{2}(BL + CN)(LN)$

$$= \frac{1}{2}(y_2 + y_1)(x_1 - x_2) + \frac{1}{2}(y_1 + y_3)(x_3 - x_1) - \frac{1}{2}(y_2 + y_3)(x_3 - x_2)$$

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Illustration 18

The vertices of Δ ABC are (–2, 1), (5, 4) and (2, – 3) respectively. Find the area of triangle.

Solution

A (–2, 1), B (5, 4) and C (2, – 3) be the vertices of triangle.

So, $x_1 = -2$, $y_1 = 1$; $x_2 = 5$, $y_2 = 4$; $x_3 = 2$, $y_3 = -3$

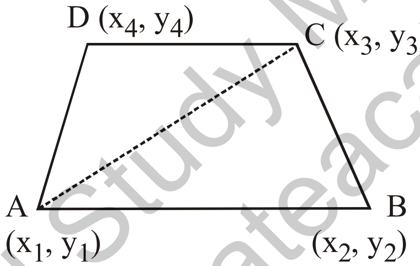
$$\text{Area of } \Delta ABC = \frac{1}{2}\left|\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right|$$

$$= \frac{1}{2}\left|\left[(-2)(4+3) + (5)(-3-1) + 2(1-4)\right]\right|$$

$$= \frac{1}{2}\left|\left[-14 + (-20) + (-6)\right]\right| = \frac{1}{2}\left|-40\right| = 20 \text{ Sq. unit.}$$

3.11 AREA OF A QUADRILATERAL:

Let the vertices of quadrilateral ABCD are A (x_1, y_1), B (x_2, y_2), C (x_3, y_3) and D (x_4, y_4).



1. Area of quadrialeral ABCD = Area of ΔABC + Area of ΔACD
2. Area of a quadrilateral with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) taken in order is given by

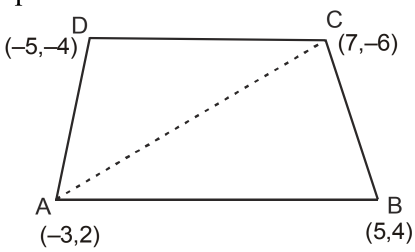
$$\frac{1}{2}\begin{vmatrix} x_1 - x_3 & y_1 - y_3 \\ x_2 - x_4 & y_2 - y_4 \end{vmatrix}.$$

Illustration 19

Find the area of quadrilateral whose vertices, taken in order, are A(–3, 2), B(5, 4), C (7, – 6) and D (–5, –4).

Solution

Area of quadrilateral = Area of ΔABC + Area of Δ ACD



So, Area of $\Delta ABC = \frac{1}{2}\left|(-3)(4+6)+5(-6-2)+7(2-4)\right|$

$= \frac{1}{2}\left|-30-40-14\right|$

$= \frac{1}{2}\left|-84\right| = 42$ Sq. units

Area of Δ ACD $= \frac{1}{2}\left|-3(-6+4)+7(-4-2)+(-5)(2+6)\right|$

$= \frac{1}{2}\left|+6-42-40\right|$

$= \frac{1}{2}\left|-76\right| = 38$ Sq. units

So, Area of quadrilateral ABCD = 42 + 38 = 80 Sq. units.

Illustration 20.

Find the fourth vertex of the parallelogram whose three consecutive vertices are (8, 8), (6, 1) and (– 1, 1).

Solution:

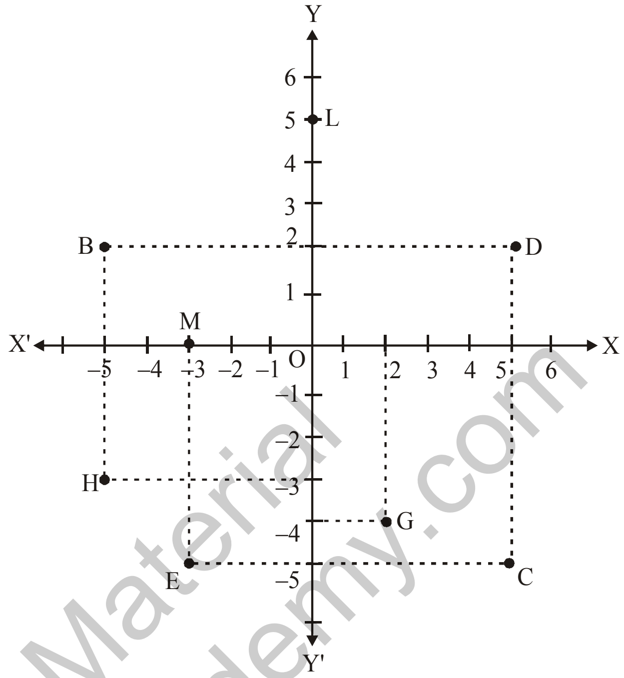
Let the three vertices of the parallelogram be A (8, 8), B (6, 1) and C (– 1, 1) then fourth vertex D (x, y) is given by

D (x, y) = $(x_1 - x_2 + x_3, y_1 - y_2 + y_3)$ = (8 – 6 – 1, 8 – 1 + 1) = (1, 8)

Hence, the fourth vertex is D (1, 8).

CONCEPT APPLICATION LEVEL - I

Q.1 See figure and write the following :



(i) The coordinates of B.

(ii) The coordinates of C.

(iii) The point identified by the coordinates (–3, –5)

(iv) The point identified by the coordinates (2, –4).

(v) The abscissa of the point D.

(vi) The coordinates of the point L.

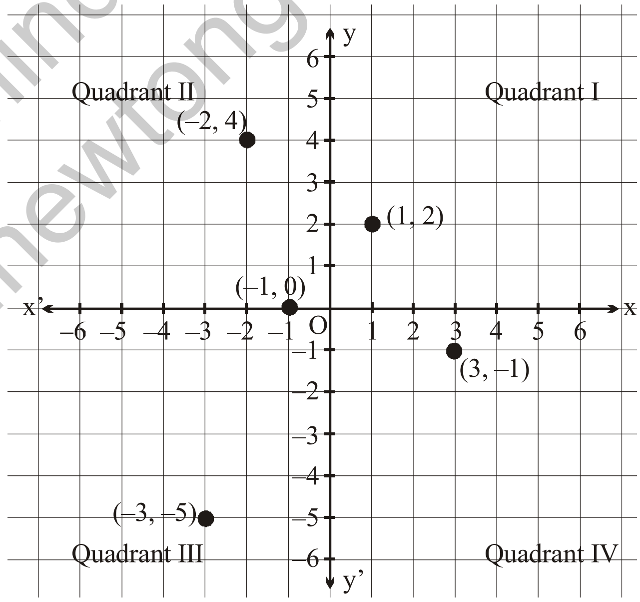
(vii) The ordinate of the point H.

(viii) The coordinates of the point M.

Sol. (i) (–5, 2) (ii) (5, – 5) (iii) E (iv) G (v) 5 (vi) (0, 5) (vii) – 3 (viii) (–3, 0)

Q.2 In which quadrant or on which axis do each of the points (–2, 4), (3, –1), (–1, 0), (1, 2) and (–3, –5) lie ? Verify your answer by locating them on the Cartesian plane.

Sol.

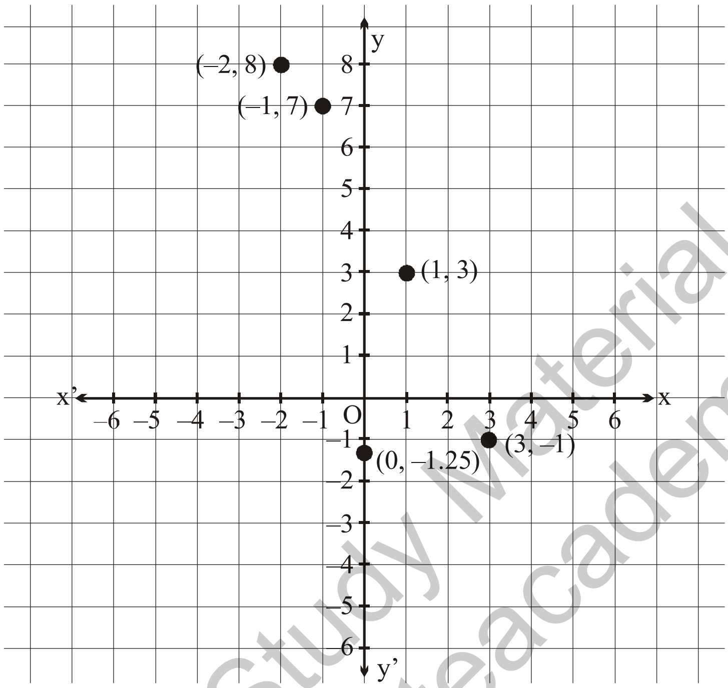


The point (–2, 4) lies in quadrant II, the point (3, –1) lies in the quadrant (IV). The point (–1, 0) lies on the negative x-axis, the point (1, 2) lies in the quadrant I and the point (–3,– 5) lies in the quadrant III.

Q.3 Plot the points (x, y) given in the following table on the plane, choosing suitable units of distance on the axes.

x	–2	–1	0	1	3
y	8	7	–1.25	3	–1

Sol.



Q.4 Write the answer of each of the following questions.

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane ?

(ii) What is the name of each part of the plane formed by these two lines ?

(iii) Write the name of the point where these two lines intersect.

Sol. (i) Horizontal line is called x-axis and vertical line is called y-axis.

(ii) Quadrants

(iii) Origin

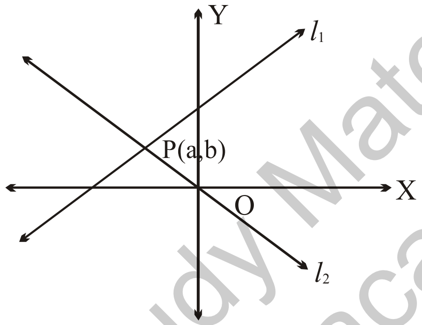
CONCEPT APPLICATION LEVEL - II

Q.1 What is the quadrilateral that is formed by joining the points (1, 1), (2, 4), (8, 4) and (10, 1)?
(A) A triangle (B) A square (C) A rectangle (D) A trapezium

Q.2 The graph of the linear equation 3x + 2y = 6 cuts the y-axis at the point
(A) (2, 0) (B) (0, 2) (C) (0, 3) (D) (3, 0)

Q.3 Abscissa of all the points on x-axis is __________.
(A) 0 (B) 1 (C) 2 (D) any number

Q.4 In the adjoining figure, point P the intersection of lines l_1 and l_2 has the coordinates (a, b) as shown here. Which of the following could be the ordered pair (a, b)?



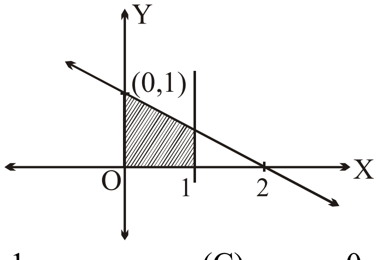
(A) $\left(5, \frac{-5}{2}\right)$ (B) $\left(-1, \frac{5}{8}\right)$ (C) (0, 4) (D) $\left(\frac{2}{3}, \frac{1}{3}\right)$

Q.5 Consider the point A (a, b + c), B (b, c + a) and C (c, a + b). The area of ΔABC is :
(A) $2(a^2 + b^2 + c^2)$ (B) $\frac{a^2 + b^2 + c^2}{6}$ (C) 2(ab + bc + ca) (D) None of these

Q.6 A straight line parallel to the x-axis has equation
(A) x = a (B) y = a (C) y = x (D) y = –x

Q.7 Find the equation of the line parallel to 4x + 3y = 5 and having x-intercept (– 3).
(A) 3x + 4y + 12 = 0 (B) 3x + 4y = 12 (C) 4x + 3y –12 = 0 (D) 4x + 3y + 12 = 0

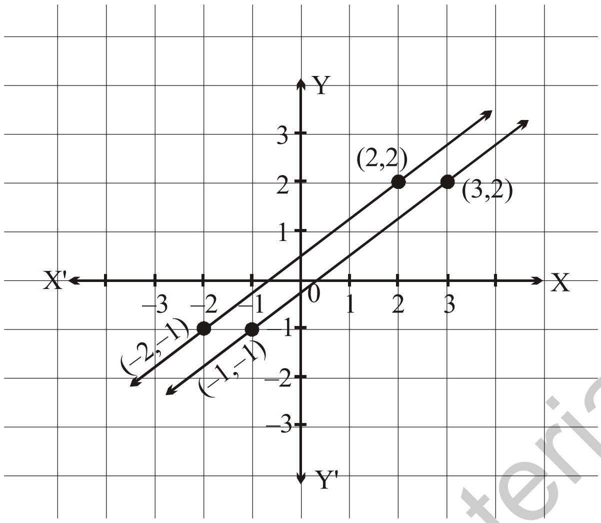
Q.8 In the rectangular coordinate system below, the shaded region is bounded by straight lines. Which of the following is not an equation of one of the boundary lines?



(A) x = 0 (B) x = 1 (C) x – y = 0 (D) x + 2 y = 2

Q.9 The Cartesian system is named in honour of the mathematician __________.
(A) Lesbnitz (B) Euclid (C) Laplace (D) Rene Descartes

Q.10 The equation representing the given graph is :



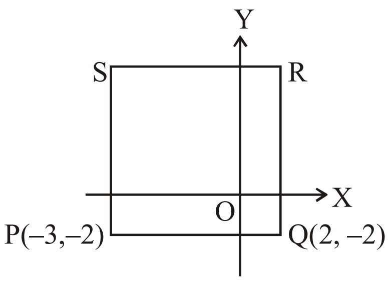
(A) $7x + 2y = 11; y - 2x = 3$ (B) $2x + 7y = 11; 5x + \frac{35y}{2} = 25$
(C) $3x - 7y = 10; 8y - 6x = 4$ (D) $3x - 4y = 1; 8y - 6x = 4$

Q.11 In the xy-plane let A be the point (5,0) and L be the line $y = \frac{x}{3}$. The number of poins P on the line L such that triangle OAP is isosceles is (O being the origin)
(A) 2 (B) 3 (C) 4 (D) 5

Q.12 Ordered pair(s) that satisfy the inequation $x + y + 1 < 0$, is
(A) (0, –1) (B) (–2, 0) (C) (2, –4) (D) Both (B) and (C)

Q.13 The area of the triangle formed by $2x + 3y = 6$ and the coordinate axes is _________.
(A) 3 sq. units (B) 2 sq. units (C) 6 sq. units (D) 5 sq. units

Q.14 The given diagram is drawn on a cartesian plane



PQRS is a square. The coordinates of S are
(A) (–3, 3) (B) (3, –3) (C) (–3, –3) (D) (–3, 2)

Q.15 Which option is correct with respect to the line $x + 1 = 0$?
(A) It is parallel to y-axis (B) It passes through (0, –1)
(C) It is parallel to x-axis (D) It passes through (0, 0)

CONCEPT APPLICATION LEVEL - III

SECTION-A

- **Multiple choice question with one correct answer**

Q.1 If two opposite vertices of a square are (5, 4) and (1, –6), then the coordinates of its remaining two vertices is :

(A) (–2, 2) & (5, 3) (B) (8, –3) & (–2, 1) (C) (8, 6) & (3, 5) (D) (1, –3) & (2, 5)

Q.2 The distance of the point (2, 0) from the origin is

(A) 1 unit (B) 4 units (C) 2 units (D) none of these

Q.3 If coordinates of point P are (x, y) such that $x < 0$, $y = 0$, then P lies on

(A) positive x-axis (B) positive y-axis (C) negative y-axis (D) negative x-axis

Q.4 If two vertices of an isosceles triangle are (2, 0) and (2, 5) and length of the equal sides is 3, then the third vertex is :

(A) (2, 6) & (– 5, 3) (B) (8, 3) & (5, 1) (C) $(2 \pm \frac{\sqrt{11}}{2}, \frac{5}{2})$ (D) $(3 \pm \frac{\sqrt{14}}{2}, \frac{7}{2})$

Q.5 If the point (0, 2) is equidistant from the points (3, k) and (k, 5), then the value of k is :

(A) 0 (B) 2 (C) – 2 (D) None of these

Q.6 To find the abscissa of a point in the first quadrant, which of the given steps is correct:

(A) Join the point with the x-axis

(B) Draw perpendicular on the x-axis from the point.

(C) Find the perpendicular distance of that point with the y-axis.

(D) None of these

Q.7 To locate a point (–a, –b) in the third quadrant if $a > 0$, $b > 0$.

(A) Move only in the negative direction of x-axis.

(B) Move ‘a’ units in the negative direction of x-axis then b units vertically downward in the negative direction of y-axis parallel to the y-axis.

(C) Move ‘b’ units along the x-axis

(D) None of these.

Q.8 If the distance between the points (a, 2) and (3, 4) be 8 then a =

(A) $2+3\sqrt{15}$ (B) $2-3\sqrt{15}$ (C) $2\pm3\sqrt{15}$ (D) $3\pm2\sqrt{15}$

Q.9 To find the ordinate of a point in the first quadrant.

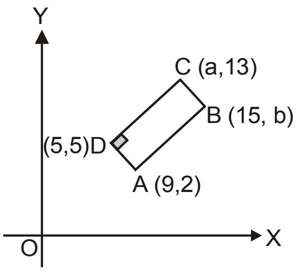
(A) Find the perpendicular distance of the point from the x-axis.

(B) Find the distance of the point from the y-axis.

(C) Find the distance of the point from the origin.

(D) None of these.

Q.10 In the rectangle shown, the value of a – b is :



(A) – 3 (B) – 1 (C) 3 (D) 1

Q.11 The co-ordinates of one end of a diameter of a circle are (5 , –7) . If the co-ordinates of the centre be (7 , 3) , the co-ordinates of the other end of the diameter are :

(A) (6 , –2) (B) (9 , 13) (C) (–2 , 6) (D) (13 , 9)

Q.12 The point (11 , 10) divides the line segment joining the points (5 , –2) and (9 , 6) in the ratio :

(A) 1 : 3 internally (B) 1 : 3 externally (C) 3 : 1 internally (D) 3 : 1 externally

Q.13 If A & B are the points (–3, 4) and (2, 1), then the co-ordinates of the point C on produced AB such that AC = 2 BC are :

(A) (2, 4) (B) (3, 7) (C) (7, –2) (D) $\left(-\frac{1}{2},\frac{5}{2}\right)$

Q.14 If the three vertices of a parallelogram are (a + b, a – b), (2a + b, 2a – b) and (a – b, a + b), then the fourth vertex is :

(A) (– a, a) (B) (– a, – a) (C) (– b, – b) (D) None

Q.15 If the middle points of the sides of a triangle be (–2, 3), (4, – 3) and (4, 5), then centroid of triangle is :

(A) $\left(\frac{5}{3},2\right)$ (B) $\left(\frac{5}{6},1\right)$ (C) $\left(1,\frac{5}{6}\right)$ (D) $\left(2,\frac{5}{3}\right)$

Q.16 The orthocentre of the triangle ABC is 'B' and the circumcentre is 'S' (a, b). If A is the origin then the co-ordinates of C are :

(A) (2a, 2b) (B) $\left(\frac{a}{2},\frac{b}{2}\right)$ (C) $\left(\sqrt{a^2+b^2},\ 0\right)$ (D) None

Q.17. If the centroid and circumcenter of a triangle are (3, 3) and (6, 2) respectively, then the orthocentre is :
(A) (–3, 5) (B) (–3, 1) (C) (3, –1) (D) (9, 5)

Q.18 Orthocentre of triangle with vertices (0, 0), (3, 4) and (4, 0) is :
(A) $\left(-3, \frac{3}{4}\right)$ (B) (3, 12) (C) $\left(3, \frac{3}{4}\right)$ (D) (3, 9)

Q.19 If the points A (6, 1), B (8, 2), C (9, 4) and D (P, 3) are the vertices of a parallelogram taken in order. Then the value of P is :
(A) 7 (B) 8 (C) 4 (D) 9

Q.20 The area of the triangle whose vertices are (a,a), (a + 1, a + 1) and (a +2, a) is :
(A) a^3 (B) 1 (C) 2a (D) $2^{1/2}$

Q.21 The points (– a, – b), (0, 0), (a, b) and (a^2, ab) are :
(A) Collinear (B) Vertices of a parallelogram
(C) Vertices of a rectangle (D) None of these.

Q.22 If the co-ordinates of two points A and B are (3, 4) and (5, –2) respectively, then the co-ordinates of any point P if PA = PB and Area of ΔPAB = 10 is :
(A) (7, 2) or (1, 0) (B) (– 7, 2) or (3, 0) (C) (7, – 2) or (5, 0) (D) (7, –2) or (–1, 0)

SECTION-B

- **Multiple choice question with one or more than one correct answers**

Q.1 If the coordinates of point A are (a, b) with ab > 0, then A lies in
(A) I quadrant (B) II quadrant (C) III quadrant (D) IV quadrant

Q.2 Which of the following statements is/are correct.
(A) The coordinate axes are mutually perpendicular to each other.
(B) The y-axis is also called as ordinate.
(C) The point of intersection of x-axis and y-axis is called as origin.
(D) The x-axis is also called as abscissa.

Q.3 The perpendicular distance of point P(x, y) from y-axis is known as
(A) x coordinate (B) ordinate (C) y-coordinate (D) abscissa

Q.4 The perpendicular distance of point P(x, y) from x-axis is known as
(A) x-coordinate (B) abscissa (C) ordinate (D) y-coordinate

SECTION-C

- **Comprehension**

Plot the points A(–2, 0) B(2, 0), C(2, 2), D(0, 4), E(–2, 2) on the graph paper and join them in order. Now answer the following questions according to the figure obtained.

Q.1 What is the figure obtained by joining the points ABCDE.
(A) square (B) rectangle (C) triangle (D) pentagon

Q.2 What is the area of the figure so formed.
(A) 12 sq. units (B) 8 sq. units (C) 4 sq. units (D) 2 sq. units

Q.3 What is the distance of point E from x-axis.
(A) –2 units (B) 4 units (C) –4 units (D) 2 units

SECTION-D

- **Assertion & Reason**

Instructions: In the following questions as Assertion (A) is given followed by a Reason (R). Mark your responses from the following options.
(A) Both Assertion and Reason are true and Reason is the correct explanation of 'Assertion'
(B) Both Assertion and Reason are true and Reason is not the correct explanation of 'Assertion'
(C) Assertion is true but Reason is false
(D) Assertion is false but Reason is true

Q.1 **Assertion:** Point P(–2, 3) is at a distance of 3 units from x-axis.
Reason: Ordinate gives the perpendicular distance of point from x-axis.

Q.2 **Assertion:** The distance of point R(3, 4) from origin is 3 units.
Reason: The distance between two points is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

SECTION-E

- **Match the following (one to one)**

Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. Only one entry of column-I may have the matching with the one entry of column-II and one entry of column-II may have only one matching with one entry of column-I

Q.1

	Column I [Point (x, y)]		Column II [Point (x, y) lines in]
(i)	$x > 0, y > 0$	(P)	Q_2
(ii)	$x < 0, y > 0$	(Q)	Q_1
(iii)	$x < 0, y < 0$	(R)	Q_4
(iv)	$x > 0, y < 0$	(S)	Q_3

Q.2

	Column I [Point (x, y)]		Column II [Point (x, y) lines on]
(i)	$x > 0, y = 0$	(P)	– ive x-axis
(ii)	$x < 0, y = 0$	(Q)	– ive y-axis
(iii)	$x = 0, y < 0$	(R)	+ ive y-axis
(iv)	$x = 0, y > 0$	(S)	+ ive x-axis

SECTION-F

- **Match the following (one to many)**

  Column-I and **column-II** contains **four** entries each. Entries of column-I are to be matched with some entries of column-II. One or more than one entries of column-I may have the matching with the same entries of column-II and one entry of column-II may have one or more than one matching with entries of column-I

Q.1

	Column I [Point P(a, b)]		Column II (Quadrant)
(i)	$a > 0, b \geq 0$	(P)	III quadrant
(ii)	$a < 0, b < 0$	(Q)	I quadrant
(iii)	$a > 0, b = 0$	(R)	Positive x-axis
(iv)	$a < 0, b \leq 0$	(S)	Negative x-axis

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

Q.1	D	Q.2	C	Q.3	D	Q.4	B	Q.5	D	Q.6	B	Q.7	A
Q.8	C	Q.9	D	Q.10	D	Q.11	C	Q.12	D	Q.13	A	Q.14	A
Q.15	A												

CONCEPT APPLICATION LEVEL - III

SECTION-A

Q.1	B	Q.2	C	Q.3	D	Q.4	C	Q.5	D	Q.6	C	Q.7	B
Q.8	D	Q.9	A	Q.10	D	Q.11	B	Q.12	D	Q.13	C	Q.14	D
Q.15	D	Q.16	A	Q.17	A	Q.18	C	Q.19	A	Q.20	B	Q.21	B
Q.22	A												

SECTION-B

Q.1	AC	Q.2	AC	Q.3	AD	Q.4	CD

SECTION-C

Q.1	D	Q.2	A	Q.3	D

SECTION-D

Q.1	A	Q.2	D

SECTION-E

Q.1 (i)-(Q), (ii)-(P), (iii)-(S), (iv)-(R)

Q.2 (i)-(S), (ii)-(P), (iii)-(Q), (iv)-(R)

SECTION-F

Q.1 (i)-(Q,R),(ii)-(P), (iii)-(R), (iv)-(P,S)